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# A MODEL OF LABOUR SUPPLY WITH JOB OFFER RESTRICTIONS

by

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## Abstract

We specify a model of labour supply with job offer restrictions. A job offer is defined by a wage rate and working hours. The number of job offers is restricted, and follows a Poisson distribution. Individuals choose the alternative with the higher utility level. Three specifications are estimated: A basic specification, a specification in which the number of job offers depends on characteristics, and a specification in which the wage rate depends on working hours. Although the properties of the sample distribution of hours observed are replicated well for all of the specifications, the implications for the underlying behaviour are different.

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# 1 Introduction

In the mid-eighties it was recognized that the standard neo-classical labour supply model (see e.g. Heckman (1974), Hausman (1980)), was not able to explain the patterns observed in the empirical distribution of weekly working hours. In particular, it is unable to explain the high peaks at levels like 40 hours a week. Dickens and Lundberg (1985) introduced hours restrictions into the labour supply model assuming that hours arrive from a discrete offer distribution. Tummers and Woittiez (1991) extended the model by making the wage rate dependent on hours. Van Soest, Woittiez and Kapteyn (1990) compared the standard model with the model with hours restrictions using a Dutch data set on labour supply, taking into account the tax system, and their estimation results are relevant empirical evidence in favour of the hours restrictions model. In Blundell, Ham and Meghir (1987) a different approach to restrictions on the labour market is followed. Here the emphasis is on the modelling of involuntary unemployment.

Following Tummers and Woittiez (1991), and Van Soest, Woittiez and Kapteyn (1990), we specify a model of labour supply with job offer restrictions: Individuals are restricted in their choice of working hours by the limited availability of jobs. In the previous studies, job offers depend on hours only, but not on the wage. This is in contrast with job search models (see Mortensen (1986) for an overview), in which job offers are characterized by wages, arriving from a wage offer distribution.

We thus make the assumption that a job offer consists of two characteristics, a wage component and an hours component. An individual chooses the

job yielding the highest utility level. If all jobs offered generate utility levels less than the utility of not working, the individual prefers to be jobless. It is possible that an individual will receive no job offer at all, so involuntary unemployment may arise.

We compare three different model specifications: A basic model and two extensions. The emphasis is on comparing the implications of the different specifications. The distribution that generates the number of job offers, is allowed to depend on individual characteristics, and we test whether the average number of job offers depends on individual characteristics. Moreover, the model is estimated both with and without hours dependence of the wage equation, which enables us to test for the significance of hours in the wage equation. Because of the flexible specification of the distribution of hours offered, the distribution of weekly working hours looks similar to the empirical distribution for all of the three specifications. However, the different specifications turn out to have quite different implications for the underlying behaviour. For example, the estimates of the specification without hours in the wage equation show that few jobs with weekly working hours above 40 are offered, whereas the model in which the wage equation includes hours suggests that individual choice is the reason for observing low frequencies of weekly working hours above the full time level.

We basically use the same data as Tummers and Woittiez (1991) and Van Soest, Woittiez and Kapteyn (1990).

In section 2 the model is presented. Section 3 includes the likelihood contribution for estimating the model parameters by maximum likelihood.

Section 4 presents the data, as well as the estimation results for various model specifications. Simulation experiments are performed. Section 5 concludes.

## 2 The model

Individuals are constrained in their choice of labour by the availability of job offers. The number of job offers,  $n$ , follows a Poisson distribution with parameter  $\lambda$ . An advantage of this Poisson specification, as opposed to the binomial specification of Tummers and Woittiez (1991) and Van Soest et al. (1990), is that it can easily be made dependent on individual characteristics. In these studies, the effect of individual characteristics have typically been ignored. Moreover, the binomial distribution requires the choice of a fixed maximum number of offers.

A job offer has two determinants, the wage  $w$  and a weekly number of working hours  $h$ . A job offer is modelled as a simultaneous draw of a wage rate  $w$  and a weekly number of working hours  $h$  from a joint wage-hours offer distribution  $f(w, h)$ . Hours are assumed to be distributed according to a discrete distribution <sup>1</sup>:

$$P(h = h_l) = p_l, l = 1, \dots, m \quad (2.1)$$

in which  $m$  is the number of categories of hours, and  $h_l, l = 1, \dots, m$ , are the different values that the hours offer can take. The advantage of this approach is that no heavy restrictions, like single peakedness or symmetry, are placed on the shape of the distribution. The wage offer distribution is assumed to

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<sup>1</sup>A discrete hours distribution was introduced by Dickens and Lundberg (1985).

be log-normal:

$$\ln w = x_l' \eta + v, v \sim N(0, \sigma_v^2) \quad (2.2)$$

The wage depends on individual characteristics  $x_l$ . The subscript  $l$  indicates possible dependence on the hours offer. Tummers and Woittiez (1991) specified a normal wage distribution, thereby not restricting the range of possible wages to positive values. The equation is formulated in terms of the net wage rate.

Individuals have a utility function  $u(h, y)$ , defined over labour supply  $h$  and income  $y$ . The utility function is specified according to Hausman (1980). The Hausman (1980) utility function yields a linear labour supply function when maximized subject to a linear budget constraint.

$$u(h, y) = -\ln(\gamma - \beta h) - \frac{\beta(h - X\delta - e - \beta y)}{\gamma - \beta h} \quad (2.3)$$

where

- $\beta, \gamma, \delta$  are parameters,  $\beta < 0, \gamma > 0$
- $y$  is disposable income
- $h$  is the number of working hours
- $e$  is an unobserved random taste variable,  $e \sim N(0, \sigma_e^2)$
- $X$  is a vector of individual characteristics

For any given number of working hours the budget constraint is <sup>2</sup>

$$y = wh + \mu \quad (2.4)$$

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<sup>2</sup>Non-linearity in hours arises if the wage rate  $w$  depends on hours.

where  $\mu$  is non-labour income.

At a given point in time, an individual receives  $n$  (possibly zero) job offers, each of them consisting of a wage  $w, 0 < w < \infty$  and a number of working hours  $h \in \{h_1, \dots, h_m\}$ . Furthermore, an individual can always choose not to work. The alternative which yields the highest level of utility will be chosen. An individual will be observed to be non-working if the utility level of not working exceeds the utility level of all of the  $n$  job offers.

### 3 Maximum likelihood estimation

The model parameters will be estimated by maximum likelihood. For working individuals, let the observed wage-hours pair be denoted by  $(w_*, h_{l_*}), l_* \in \{0, 1, \dots, m\}$ , and  $u_0$  denotes utility at zero hours. In the appendix it is shown that the likelihood contribution of a working individual, given the random preferences  $e$ , is

$$l(w_*, h_{l_*} | e) = \lambda \exp\{-\lambda[1 - P(w_*, h_{l_*} | e)]\} k(w_*, \eta' x_l, \sigma_v) p_{l_*} \quad (3.1)$$

if  $u(h_{l_*}, w_* h_{l_*} + \mu) > u_0$ , zero otherwise

The function  $k(\cdot)$  denotes the log-normal wage offer density function.  $P(w_*, h_{l_*} | e)$  is the probability that the observed wage-hours combination has a utility level at least as high as that of any other possible combination of wages and hours<sup>3</sup>. Let  $g_l(e)$  denote the wage at which the individual is indifferent between the observed combination  $(w_*, h_{l_*})$  and working hours of  $h_l$ , at random preferences  $e$ , i.e.  $u(h_l, g_l(e)h_l + \mu) = u(h_{l_*}, w_* h_{l_*} + \mu)$ . Then  $P(w_*, h_{l_*} | e)$  can be

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<sup>3</sup>Derivations and further details can be found in the appendix.

written as

$$P(w_*, h_{l_*} | e) := \sum_{l=1}^m p_l P(w \leq g_l(e) | e) \quad (3.2)$$

This is the probability that any other combination of wages and hours  $h_l, l = 1, \dots, m$ , has a wage at most as high as its indifference wage  $g_l(e)$ . If wages follow the log-normal distribution, the probability can be expressed in terms of the standard normal distribution function  $\Phi(\cdot)$  :

$$P(w \leq g_l(e) | e) = \Phi\left(\frac{\ln g_l(e) - \eta' x_l}{\sigma_v}\right) \quad \text{if } g_l(e) > 0 \quad (3.3)$$

$$= 0 \quad \text{if } g_l(e) \leq 0 \quad (3.4)$$

For non-working individuals, the conditional likelihood function is

$$l(h = 0 | e) = \exp\{-\lambda[1 - P(0|e)]\} \quad (3.5)$$

with  $P(0|e)$  as in (3.2), evaluated at zero hours, being the probability that the utility of not working is at least as high as the utility level of an arbitrary job offer.

After integrating over random preferences, the likelihood contribution for workers is

$$l(w_*, h_{l_*}) = \int_B l(w_*, h_{l_*} | e) \frac{1}{\sigma_e} \phi\left(\frac{e}{\sigma_e}\right) de, l_* \in \{1, \dots, m\}, 0 < w < \infty \quad (3.6)$$

$\phi(\cdot)$  being the standard normal density function, and  $B$  being the region of random preferences in which utility of not-working is lower than the utility of the observed wage-hours combination:

$$B := \{e | u_0 \leq u(h_{l_*}, w_* h_{l_*} + \mu)\} \quad (3.7)$$



**Table 1: Sample statistics, Sample Size = 849**

Variable	Mean	Stand. dev.
Hours : weekly working hours	27.3	12.5
Net Wage Rate : guilders/hour	12.5	4.5
Family size : persons	3.5	1.2
Age	37.2	9.9
Non-labor income (guilders/week)	713.5	301.3
Variable	Number of observations	
Labor market state: Number of employed	331	
Educ1	217	
Educ2	223	
Educ3	322	
Educ4	71	
Children with age below 6	221	
Sector 1	4	
Sector 2	121	

For non-workers, the likelihood contribution is

$$l(h = 0) = \int \exp\{-\lambda[1 - P(0|e)]\} \frac{1}{\sigma_e} \phi\left(\frac{e}{\sigma_e}\right) de \quad (3.8)$$

## 4 Empirical results

### The data

The sample is a cross section data set from OSA <sup>4</sup>. It is a sample of 849 married females, drawn from the Dutch population in 1985. Table 1 contains sample statistics. There are five levels of education. The highest (educ5), serves as reference level. Moreover, three types of education are distinguished: non-technical and non-commercial type of education (sec1), semi-technical and semi-commercial type of education (sec2), and technical

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<sup>4</sup>Organisatie voor Strategisch Arbeidsmarktonderzoek (Organization for Strategic Labor Market Research; a government agency in the Netherlands).

and commercial type of education, the reference level.

## Estimation results

The number of hours categories  $m$  in the hours offer distribution (2.1) is chosen to be 15, and hours are classified in groups of four:

$$P(h = h_l) = p_l \text{ with } h_l = 4 \times l, l = 1, \dots, m \quad (4.1)$$

As a result, the maximum number of hours that can be offered is 60. This choice has been made on basis of the range of observed hours. Moreover, in the distribution of observed hours, peaks at 20, 32 and 40 hours a week occur, whereas the observed frequency of intermediate levels is low <sup>5</sup>. Therefore, some equality restrictions are placed on probabilities of hours categories with a low sample frequency. These restrictions are

$$\begin{aligned} p_1 &= p_2 = p_3 = p_4 && (1 \text{ to } 16 \text{ hours per week}) \\ p_6 &= p_7 && (24\text{-}28 \text{ hours per week}) \\ p_{12} &= p_{13} = p_{14} = p_{15} && (48 \text{ hours a week or more}) \end{aligned} \quad (4.2)$$

Three model specifications have been estimated: the basic version (number of job offers does not depend on characteristics, the wage rate does not depend on hours), a second version with the number of job offers depending on individual characteristics, and a third version with a wage equation that depends on working hours. For the second version, the Poisson distribution is parameterized as

$$\lambda_i = \exp(\theta' z_i), i = 1, \dots, N \quad (4.3)$$

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<sup>5</sup>See, e.g., figure 1.

The wage equation for the third specification depends on hours, by specifying the wage equation

$$\ln w = \eta'x + \tau_1 h + \tau_2 h^2 + v \quad (4.4)$$

The equation is formulated in terms of the net wage rate. Barzel (1973) argues that for the gross wage rate, lower wages will be paid at low hours of work, due to fixed cost of work for the firm, whereas at sufficiently high hours of work, the marginal wage rate will be lower as well, due to decreasing marginal productivity. This suggests an inverted U-shaped relation between the wage rate and hours. If the wage equation were formulated in terms of levels, in which case the model allows for negative values of the wage rate, Moffit (1984) shows that this hypothesis leads to an S-shaped budget constraint.

Another reason for the wage rate to depend on hours is that at institutionally determined numbers of hours like 20, 32 and 40, the offered wages may be higher, and therefore, hours restrictions need not be the only explanation for observing peak levels in the empirical distribution of working hours. Including dummy variable for these peak levels in the wage equation allows for testing the latter hypothesis. However, inspection of the data led us to abstain from it because no such relation appears to be present.

Alternatively, the appearance of hours in the wage equation may be caused by the absence of any demand side conditions, like unemployment rates. Times of low unemployment possibly go together with both higher wages and larger proportions of people working full time. To be able to separate this relation between hours and wages from other effects, business cycle

variables, like the (regional) unemployment rate or vacancies in the economy, could be included in the model. The importance of business cycle variables in labour supply models is emphasized in a study by Blundell, Ham and Meghir (1998). The present paper, however, is based on a single cross section data set. Identification of business cycle effects in a cross section would have to be based on regional differences in variables like the unemployment rate. Whether a country like the Netherlands is large enough for regional differences in the unemployment rate to cause differences in the wage level is questionable. The availability of data over several years instead of a single cross section would be desirable, if not necessary, for the identification of business cycle effects.

Table 2 shows the estimation results for the three specifications. Family size and the number of children younger than 6, influence the preference for working negatively. Note that the size of the estimates of the preference parameters is different for the three specifications. For the second specification, the (absolute) values of the estimates of the preference parameters, including that of the standard deviation of random preferences, are rather large, and moreover, they are imprecisely determined. The standard deviation of the distribution of random preferences is lowest in the third specification.

The estimate of the Poisson parameter  $\lambda$  for the third specification is about half the value of the estimate of the basic model. For the basic model, the estimate value of 36.7 is rather high. A possible reason for this is that both variance and mean of the Poisson distribution are determined by the same constant parameter  $\lambda$ . The second specification, which allows  $\lambda$  to

depend on characteristics, gives the model more flexibility in this respect. Indeed, the value of  $\lambda_i$ , evaluated at sample averages of the characteristics, is 1.86. In Tummers and Woittiez (1991), the fixed maximum number of job offers of the binomial distribution that they specified, was set equal to 10. The present results show that choosing a fixed, a priori maximum number of job offers in a one parameter distribution may place too heavy a restriction on the specification. The likelihood ratio test statistic, for testing the null whether  $\lambda$  depends on characteristics, based on the log-likelihood value of the first two specifications, is 83.6, which is above the critical value at the 5% level of 14.07.

For all of the three specifications, the mean of the log-wage increases with the level of education. Although the age pattern is similar in the sense that all models have a positive parameter estimate for log-age and a negative for log-age squared, the highest wage is reached at age levels of 33, 41 and 28 for the three respective models.

The estimates of the distribution of hours offered, also shows differences between specifications. The model with covariates in  $\lambda$  places higher probabilities on weekly working hours of 20 or less, compared to the basic model, and lower probabilities on higher levels of hours. The model with hours dependent wages, on the contrary, places higher probabilities on higher working hours than the basic model. In particular, the offer probabilities of weekly working hours higher than 40 are much higher. Consequently, the explanation for the low frequency of observed hours above 40 in the basic model, is that these values of hours are hardly offered. The model with hours de-

pendent wage offers however, suggests that the probability of receiving hours levels of 44 or higher is not so low at all, but the marginal increase in income as a result of working an additional hour is that low, compared to the effect on the marginal utility of leisure, that the individuals are in general not willing to supply these high levels of hours.

The likelihood ratio test statistic to test the hypothesis  $\tau_1 = \tau_2 = 0$  has the value 16.6. The critical value at the 5% level is 6.0, so the hypothesis is rejected.

## Simulations

More insight in the performance of the three models is obtained by simulating the distribution of weekly working hours. For each individual, a random preference parameter  $e$  and a number of job offers  $n$  are drawn from their assumed distributions. Next,  $n$  wage-hours pairs are drawn, the utility levels are calculated, and the highest utility level is compared with the utility of non-working to make the participation decision. This procedure is repeated 10 times. Table 3 shows the simulated frequencies for each of the three specifications. Apart from the simulated frequencies, the table presents desired frequencies. These are based on the number of working hours the individual would have chosen if she were not restricted in hours: the tangency point of the indifference curve and the budget constraint. For a linear budget constraint, the utility maximizing number of hours of utility

function (2.3) is generated by  $h$  with

$$\begin{aligned} h &= 0 && \text{if } h^* \leq 0 \\ &= h^* && \text{if } h^* > 0 \\ h^* &= \beta\mu + \gamma w + X\delta + e \end{aligned} \tag{4.5}$$

To simulate desired hours, a random preference parameter  $e$  and a number of job offers  $n$  are drawn. As the individual is not restricted in her working hours, the job offer is determined by the wage rate. Draw  $n$  wage rates and choose the highest. Simulate desired hours according to (4.5).

The participation decision, and the peaks at 20, 32 and 40 hours a week are predicted well by the three model specifications.

For the basic model, desired participation is somewhat higher than the actual participation: the frequency of observed participation is 0.390, whereas the frequency of desired participation is 0.473. Moreover, desired participation at 40 hours a week is about three times smaller than the actual participation at 40 hours a week. For the basic model, the hours distributions are plotted in figure 1 (positive hours only). The peaks at the values of 20, 32 and 40 are not present in the distribution of desired hours.

For the second specification, in which  $\lambda$  depends on characteristics, the distributions of simulated and desired hours are depicted in figure 2. For simulated hours, the figure is similar to figure 1. However, the distribution of desired hours is very flat. This is due to the large estimate of the standard error of random preferences. The flat distribution of desired hours suggests that there are equal number of people who desire hours in each category.

The distribution of simulated and desired hours for the specification with hours dependent wages are depicted in figure 3.

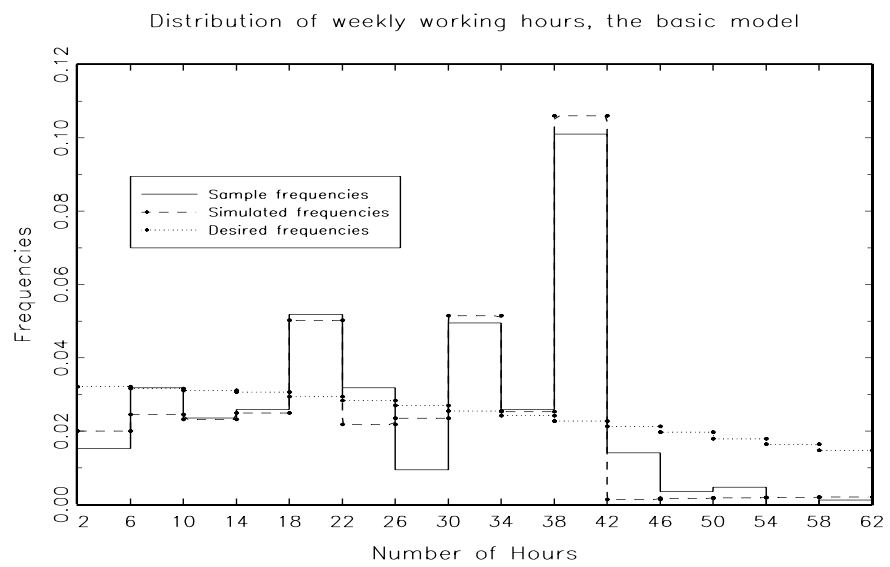


Figure 1: Distribution of working hours per week, basic model



TABLE 2 PARAMETER ESTIMATES

	Basic Model		Job offers with charact.		Hours dependent wages	
Parameters Preferences						
Parameters	Est.	St. err.	Est.	St. err.	Est.	St. err.
$\beta$	-0.029	0.0069	-0.16	0.33	-0.017	0.0048
$\gamma$	6.47	1.61	22.6	47.5	4.195	1.20
$\delta_1$ (const)	18.9	9.6	409.5	739.1	14.7	5.7
$\delta_2$ (log fs)	-56.3	10.4	-421.3	820.0	-31.6	7.9
$\delta_3$ (# child<6)	-27.2	7.4	-264.5	523.1	-16.0	5.1
$\sigma_e$	32.6	5.2	248.7	484.6	16.7	4.5
Number of Job Offers						
Parameters	Est.	St. err.	Est.	St. err.	Est.	St. err.
$\lambda$	36.7	32.3			17.6	11.0
$\theta_1$ (const)			3.8	0.55		
$\theta_2$ (log(age/17))			-3.4	0.32		
$\theta_3$ (sec1)			1.4	1.43		
$\theta_4$ (sec2)			0.40	0.17		
$\theta_5$ (educ1)			-0.57	0.53		
$\theta_6$ (educ2)			-0.97	0.52		
$\theta_7$ (educ3)			-0.45	0.48		
$\theta_8$ (educ4)			-0.12	0.51		
Parameters Wage Distribution						
Parameters	Est.	St. err.	Est.	St. err.	Est.	St. err.
$\sigma_v$	0.46	0.06	0.27	0.0011	0.36	0.04
$\eta_1$ (const)	1.45	0.29	1.86	0.12	2.22	0.19
$\eta_2$ (log(age/17))	1.37	0.24	1.91	0.26	1.39	0.24
$\eta_3$ (log(age/17)) <sup>2</sup>	-1.02	0.17	-1.1	0.20	-1.39	0.17
$\eta_4$ (educ1)	-0.48	0.07	-0.40	0.09	-0.50	0.08
$\eta_5$ (educ2)	-0.45	0.07	-0.29	0.08	-0.46	0.07
$\eta_6$ (educ3)	-0.38	0.06	-0.28	0.08	-0.36	0.07
$\eta_7$ (educ4)	-0.12	0.07	-0.10	0.08	-0.13	0.07
$\tau_1$					0.0021	0.0065
$\tau_2$					-0.00036	0.00012
Offer Distribution Hours						
Parameters	Est.	St. err.	Est.	St. err.	Est.	St. err.
$p_1 = p_2 = p_3 = p_4$	0.074	0.001	0.12	0.01	0.0076	0.0047
$p_5$	0.14	0.02	0.16	0.024	0.021	0.012
$p_6 = p_7$	0.051	0.008	0.048	0.009	0.012	0.0066
$p_8$	0.12	0.018	0.089	0.015	0.050	0.024
$p_9$	0.060	0.013	0.039	0.0092	0.043	0.018
$p_{10}$	0.23	0.028	0.13	0.021	0.31	0.10
$p_{11}$	0.032	0.010	0.015	0.0050	0.09	0.03
$p_{12} = p_{13} = p_{14} = p_{15}$	0.0056	0.0024	0.0019	0.0008	0.11	0.05
Log-likelihood value:	-2002.5		-1960.7		-1985.9	

TABLE 3 SIMULATED HOURS FREQUENCIES

		Basic Model		Job offers with charact.		Hours dependent wages	
hours	Empirical freq.	Simulated freq.	Desired freq.	Simulated freq.	Desired freq.	Simulated freq.	Desired freq.
0	0.610	0.609	0.527	0.603	0.484	0.600	0.137
4	0.0153	0.0200	0.0321	0.0177	0.00441	0.0251	0.0382
8	0.0318	0.0245	0.0316	0.0216	0.00443	0.0257	0.0498
12	0.0236	0.0232	0.0311	0.0260	0.00434	0.0250	0.0627
16	0.0259	0.0249	0.0306	0.0315	0.00432	0.0227	0.0860
20	0.0518	0.0502	0.0294	0.0529	0.00427	0.0537	0.102
24	0.0318	0.0218	0.0283	0.0188	0.00437	0.0247	0.107
28	0.00942	0.0235	0.0270	0.0228	0.00412	0.0183	0.113
32	0.0495	0.0515	0.0255	0.0509	0.00439	0.0514	0.102
36	0.0259	0.0253	0.0242	0.0272	0.00451	0.0269	0.0999
40	0.101	0.106	0.0227	0.104	0.00426	0.103	0.0729
44	0.0141	0.00132	0.0213	0.0144	0.00432	0.00142	0.0219
48	0.00353	0.00165	0.0197	0.00206	0.00432	0.00640	0.00612
52	0.00471	0.00177	0.0179	0.00221	0.00438	0.00199	0.00188
56	0.000	0.00188	0.0164	0.00241	0.00441	0.000599	0.000234
60	0.00118	0.00200	0.0147	0.00252	0.00446	0.000110	0.000
> 60			0.100		0.450		0.000

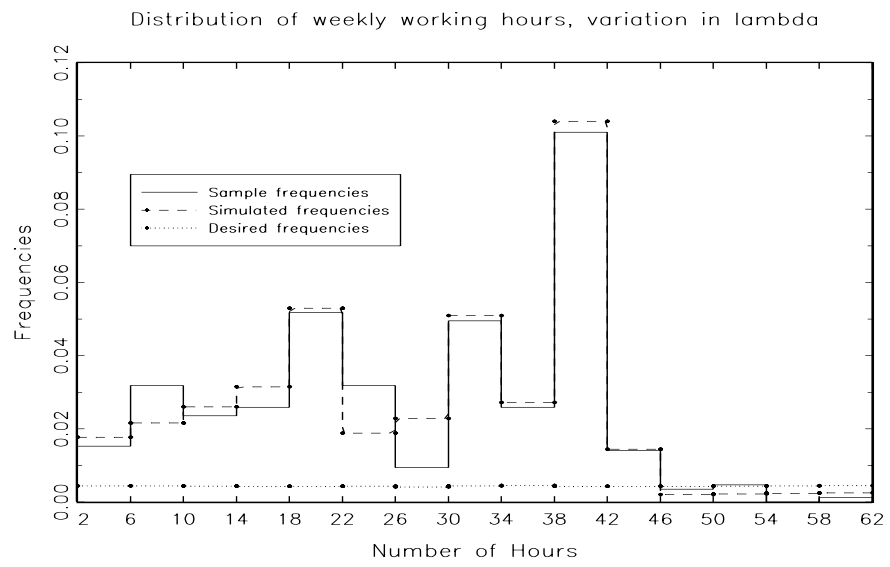


Figure 2: Distribution of working hours per week, variation in  $\lambda$

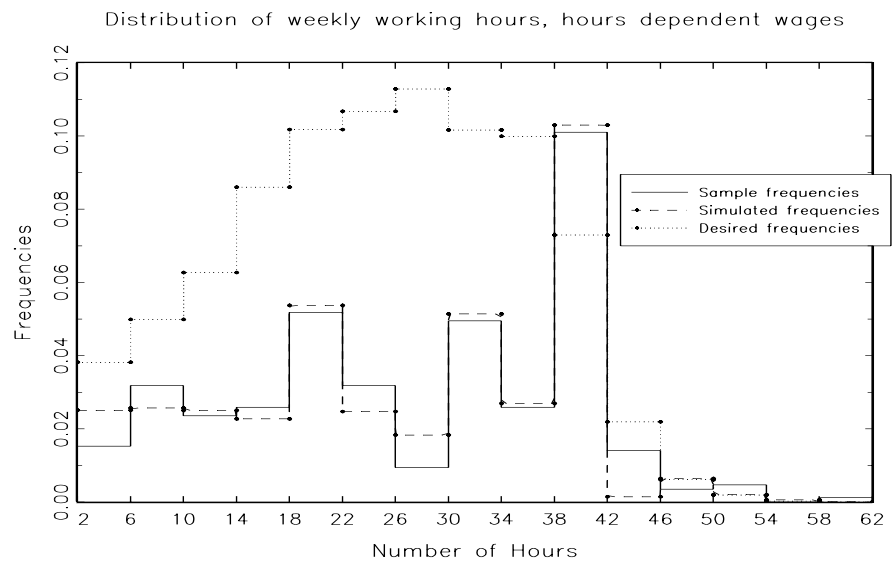


Figure 3: Distribution of working hours per week, hours dependent wages

## 5 Conclusions

Following Tummers and Woittiez (1991) and Van Soest, Woittiez and Kapteyn (1990), we specified a model of labour supply with job offer restrictions. A job offer is characterized by a wage rate and a weekly number of working hours, and the number of job offers is Poisson distributed. Individuals choose the job offer with the highest utility level. They may, however, prefer not to work, or receive no job offer.

As opposed to previous model specifications, we specify the wage equation in terms of log-wages, so that the wage distribution can generate positive wages only. Moreover, the wage is modelled as part of the job offer, conform job search theory. Instead of focussing on the difference between the neo-classical labour supply model and the model with demand side restrictions, we allow for different specifications of the model with restrictions itself. Due to the flexible functional form of the distribution of hours offered, all of the three specifications can replicate the patterns observed in the empirical distribution of working hours, like in the previous studies. However, the different specifications have quite different implications for the underlying behaviour. If the number of job offers does not depend on individual characteristics, variation in random preferences dominates the utility function. As a consequence, the distribution of desired hours is very flat. If the wage rate depends on hours, the distribution of desired hours shows a peak at 28 hours a week. For levels of hours above 40, the probabilities of hours offered are highest in the latter specification, providing a different explanation for observing low sample frequencies for these levels of hours than the other specifications.

The basic model specification is outperformed by the two extensions on basis of likelihood ratio tests. Therefore, making the number of job offers dependent on individual characteristics is a valuable extension, but the large variance in random preferences reveals that it is difficult to trace down the underlying behaviour. The message that may be obvious from this is that for an increase in the degree of 'reality' of the model we pay a price in terms of identification. The use of additional information can help us to overcome the problem of losing identification. Bloemen (1997) specifies a job search model in which individuals receive job offers over time according to a Poisson process, and decide on the acceptance of a job offer as well as on the number of working hours. The acceptance decision is characterized by a reservation wage, which depends on the individual's preferences (the utility function specified is the same as in the present paper) on the current situation and the expectation of the utility of possible future job offers. In determining this expectation the job offer arrival rate and the wage offer distribution play a role. The model contains the same sets of parameters (preference parameters, parameters of the offer distribution, the job offer arrival rate, the distribution of random preferences) as the present model, but in addition to data on accepted hours and wages, data on unemployment duration are used which enable the (nonparametric) identification of the job offer arrival rate. The hazard rate, which characterizes the distribution of unemployment duration, is defined as the product of the job offer arrival rate and the job acceptance probability (i.e. the probability that the wage offer exceeds the reservation wage).

In the literature other sources of information have been used to improve on the identification of the labour supply behaviour of individuals. Several studies make use of subjective information on the individuals' satisfaction with their labour market state or working hours. Other studies follow individuals across time and use information on the change in working hours. Ham (1982) extends the basic information on the individual's labour market state (i.e. being employed or not) with information on whether or not the individual is underemployed. Underemployed individuals are assumed to be restricted in their choice, whereas those who are not underemployed are assumed to satisfy utility maximizing behaviour in the sense that the difference between the number of hours at which the budget constraint and indifference curve are tangent and observed hours is purely random. In terms of the present paper, a different treatment of underemployed individuals may add to the identification of the model parameters: individuals who are not underemployed can be treated as in the standard neoclassical labour supply model, whereas those who are underemployed would be subject to a framework with offer restrictions. Altonji and Paxson (1986,1992) use panel data and compare changes in working hours between individuals that do and that do not change jobs. They argue that if the employer determines the working hours, the only way for individuals to get closer to their optimal hours is to change their job. In their analysis they stress the importance to correct for other job characteristics and unobservables in order to rule out that the higher variation in hours changes between jobs is spurious. Moreover, they emphasize the role that may be played by information on quits and layoffs.

Individuals that quit are more likely to be dissatisfied with working hours than individuals that are laid off. A structural model based on this idea would be a model describing the behaviour of an individual who strives for narrowing the gap between actual hours and desired hours. The availability of desired hours, or at least information on under- and overemployment would be desirable. If an individual does not manage to narrow the gap between actual hours and desired hours it can be assumed that no acceptable job offer arrived <sup>6</sup>. Euwals (1997) actually applies the analysis proposed by Altonji and Paxson (1992) using information on desired hours. Blundell, Ham and Meghir (1998) use information on search of individuals who are in the state of nonemployment. Thus they are able to classify the nonemployed as nonparticipants or participants (those searching). They show how this information enables them to decompose the probability of being a discouraged worker in the probability of being discouraged due to search costs and the probability of being discouraged due to fixed costs. They emphasize the role of demand conditions and they include various business cycle variables in the estimation of their model. Bloemen (1998) estimates a structural model of job search, including information on search. The search information can be used to obtain information on the costs of search, whereas job offer arrival rates can be identified from duration data. However, the model does not include the decision on working hours for those who accepted a job. Adding the hours decision would result in the incorporation of the model in Bloemen

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<sup>6</sup>Note that using information on job changes introduces the concept of costs of turnover (see Van den Berg (1992)) which, in turn, may require additional information for identification.



(1997).

## A Likelihood contributions

In this section, likelihood contributions for working and non-working individuals are constructed.

The number of job offers,  $n$ , follows the Poisson distribution with parameter  $\lambda$ :

$$p(n) = \frac{\exp(-\lambda)\lambda^n}{n!}, n = 0, 1, \dots, \infty \quad (\text{A.1})$$

For a working individual, the observed wage-hours combination is denoted by  $(w_*, h_{l_*})$ . If the individual received  $n$  job offers  $(w_{(j)}, h_{(j)}), j = 1, \dots, n$ , the observed job  $(w_*, h_{l_*})$  is the job with the highest utility level. Moreover, its utility level is higher than  $u_0 = u(0, \mu)$ , the utility level of not working ("the reservation utility level"). The relation between the observed job and the  $n$  offers is:

$$\begin{aligned} (w_*, h_{l_*}) = (w_{(1)}, h_{(1)}) \quad & \text{if} \quad u(h_{(1)}, w_{(1)}h_{(1)} + \mu) \geq u(h_{(j)}, w_{(j)}h_{(j)} + \mu) \\ & j = 2, \dots, n \\ & \text{and} \quad u(h_{(1)}, w_{(1)}h_{(1)} + \mu) \geq u_0 \\ (w_*, h_{l_*}) = (w_{(2)}, h_{(2)}) \quad & \text{if} \quad u(h_{(2)}, w_{(2)}h_{(2)} + \mu) \geq u(h_{(j)}, w_{(j)}h_{(j)} + \mu) \\ & j = 1, \dots, n, j \neq 2 \\ & \text{and} \quad u(h_{(2)}, w_{(2)}h_{(2)} + \mu) \geq u_0 \\ & \vdots \\ (w_*, h_{l_*}) = (w_{(n)}, h_{(n)}) \quad & \text{if} \quad u(h_{(n)}, w_{(n)}h_{(n)} + \mu) \geq u(h_{(j)}, w_{(j)}h_{(j)} + \mu) \\ & j = 1, \dots, n-1 \\ & \text{and} \quad u(h_{(n)}, w_{(n)}h_{(n)} + \mu) \geq u_0 \end{aligned} \quad (\text{A.2})$$

This scheme lists  $n$  possibilities, and for each of them, there are  $n - 1$  job offers with a utility level that does not exceed the utility level of the observed

job. The probability that the job  $(w_*, h_{l_*})$  is chosen, is obtained by  $n$  times adding up the probability, that the utility of  $(w_*, h_{l_*})$  is at least as high as  $n - 1$  other jobs:  $n[P(w_*, h_{l_*}|e)]^{n-1}$ , with  $P(w_*, h_{l_*}|e)$ , defined in (3.2). Multiplying by the appropriate wage-hours offer density, results in the likelihood contribution, conditional on random preferences  $e$  and the number of job offers  $n$ :

$$l(w_*, h_{l_*}|e, n) = n[P(w_*, h_{l_*}|e)]^{n-1} k(w_*, \eta'x, \sigma_v) p_{l_*}, l_* \in \{1, \dots, m\} \\ \text{if } u(h_{l_*}, w_* h_{l_*} + \mu) > u_0 \\ 0 \text{ otherwise} \quad (\text{A.3})$$

Multiplying (A.3) by (A.1) and summing over all the positive numbers of job offers  $n, n = 1, \dots, \infty$ , results in (3.1) the likelihood contribution, conditional on  $e$ :

$$l(w_*, h_{l_*}|e) = \lambda \exp\{-\lambda[1 - P(w_*, h_{l_*}|e)]\} k(w_*, \eta'x_l, \sigma_v) p_{l_*}, \quad l_* \in \{1, \dots, m\} \\ u(h_{l_*}, w_* h_{l_*} + \mu) > u_0 \quad (\text{A.4})$$

Integrating over random preferences, results in the likelihood contribution for working individuals (3.6).

For non working individuals, the probability is determined that not working has a higher level of utility than any of  $n$  possible job offers:

$$P(h = 0|e, n) = [P(0|e)]^n \quad (\text{A.5})$$

with  $P(0|e)$  defined below (3.5). Multiplying (A.5) by (A.1), and summing over all possible values of  $n$  yields (3.5). After integrating over random preferences, the likelihood contribution (3.8) is obtained.

## References

- Altonji, J.G. and C.H. Paxson (1986), Job Characteristics and Hours of Work, in: R.G. Ehrenberg (ed), Research in Labor Economics, Vol. 8 (JAI-press, Greenwich), 1-55.
- Altonji, J.G. and C.H. Paxson (1992), Labor Supply, Hours Constraints, and Job Mobility, Journal of Human Resources, 27, nr. 2, 256-278.
- Barzel, Y. (1973), The Determination of Daily Hours and Wages, Quarterly Journal of Economics 87, 220-238.
- Bloemen, H.G. (1997), Job Search Theory, Labour Supply and Unemployment Duration, Journal of Econometrics, 79, 305-325.
- Bloemen, H.G. (1998), Job Search, Search Intensity and Labour Market Transitions: An Empirical Analysis, working paper, Tilburg University.
- Blundell, R, J. Ham and C. Meghir (1987), Unemployment and Female Labour Supply, Economic Journal, 97, supplement conference papers, 44-64
- Blundell, R, J. Ham and C. Meghir (1998), Unemployment, Discouraged Workers and Female Labour Supply, Research in Economics, 52, 103-131.
- Dickens, W. and S. Lundberg (1985), Hours Restrictions and La-

bor Supply, NBER Working Paper

Euwals, R.W. (1997), Hours Constraints Within and Between Jobs, Center Discussion Paper no. 9764, Tilburg university.

Ham, J.C. (1982), Estimation of a Labour Supply Model with Censoring Due to Unemployment and Underemployment, Review of Economic Studies, XLIX, 335-354.

Hausman, J. (1980), The Effect of Wages, Taxes and Fixed Costs on Women's Labor Force Participation, Journal of Public Economics 14, 161-194

Heckman, J. (1974), Shadow Prices, Market Wages, and Labor Supply, Econometrica, 42, no. 4, 679-694

Moffit, R. (1984), The Estimation of a Joint Wage-hours Labour Supply Model, Journal of Labour Economics 2, 550-566

Mortensen, D.T. (1986), Job Search and Labor Market Analysis, in: O. Ashenfelter and R. Layard, eds., Handbook of Labor Economics (North-Holland, Amsterdam)

Tummers, M. and I. Woittiez (1991), A simultaneous Wage and Labour Supply Model with Hours Restrictions, Journal of Human Resources, 26, nr. 3, 393-423

Van Soest, A.H.O, I. Woittiez and A. Kapteyn (1990), Labour Supply, Income Taxes and Hours Restrictions in the Netherlands, Journal of Human Resources, 25, nr. 3, 517-558